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ON THE OPTICAL CHARACTERS OF THE VERTICAL ZONE OF AMPHIBOLES AND PYROXENES; AND ON A NEW METHOD OF DETERMINING THE EXTINCTION ANGLES OF THESE MINERALS BY MEANS OF CLEAVAGE PIECES.

By R. A. Daly.

With Three Plates.

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It is evident that, in an optical examination of monoclinic minerals with coincident optical and crystallographic planes of symmetry, the angle of extinction on (010) is a highly important datum, for it is indispensable to a knowledge of the shape and orientation of the ellipsoid of elasticity. The difficulty of preparing sections in the plane of symmetry of amphiboles and pyroxenes, and the practical impossibility of doing so in the case of very small crystals, lead the student of these species to revert to those good natural sections, cleavage plates, and inquire as to the relation subsisting between the extinction on sections parallel to (010) and that on prismatic cleavage flakes. This relation is not simple, and it was long ago demonstrated that, accepting Fresnel's optical theorem, the extinction on (110) is dependent in a complex way on the angle of extinction on (010) and on the optical angle. Michel Lévy made it clear that, for pyroxenes, the extinction on (110) would always be less than on (010), since the latter is the maximum possible value of extinction read against cleavage cracks on any section in the vertical zone. On the other hand, it was shown for the negative amphiboles that among the infinite number of possible sections made by a plane revolving in the vertical zone from (010) to (100), there is one which has the highest value of extinction in that zone, and that this value decreases as the revolving plane moves toward (010) or (100).* It is interesting to determine whether an amphibole with this property of showing a maximum of extinction for positions of the rotating plane between (010) and (100) could have an extinction-angle on (110) greater than that on

* Fouqué and Michel Lévy, Minéralogie Micrographique, p. 368.

(010). It seems to have been taken for granted by some writers that this is not true.*

It is the object of this paper to make a systematic statement of the relation between the extinction on (110) and on (010), and to indicate under what conditions the former may be used to determine the latter. First, there will be deduced a formula to express the extinction on any plane of the vertical zone when the optical angle $2V$ and ρ, the angle of extinction on (010), are known, and a graphic representation of possible extinctions in that zone will be attempted. Then specific tables will be introduced to show how $2V$, ρ, and θ' are related for any variety of pyroxene or negative amphibole, where θ' represents the extinction on a cleavage plate of that variety. Secondly, a method is proposed for the determination of ρ by means of θ' and a new angle of extinction, θ'', found after turning the cleavage piece through a certain angle about the vertical axis.

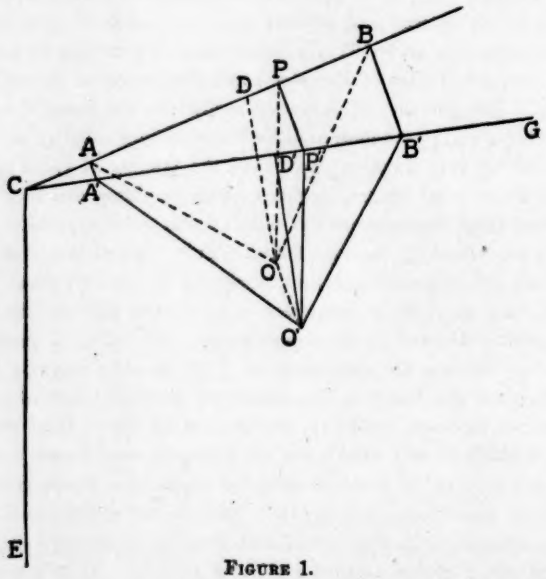

FIGURE 1.

In Figure 1, let $E\,C\,B\,O$ be the plane of symmetry of any monoclinic crystal characterized by parallelism between (010) and the optical plane. Let $E\,C\,G$ be any plane in the vertical zone cutting the plane of sym-

* Rosenbusch, Physiographie der petrographisch wichtigen Mineralien, 3te Auflage, 1892, p. 550. Zirkel, Lehrbuch der Petrographie, 2te Aufl., Bd. I. p. 302.

metry in CE, and BCG a plane cutting the vertical zone at right angles. Let OP represent the vertical crystallographic axis, OD the direction of extinction on (010), AO and BO the optic axes. Let $O'P'$ be the rectangular projection of OP on ECG, O' representing the similar projection of O on ECG. Further, let $O'D'$ be the direction of extinction on ECG, and $A'O'$ and $B'O'$ respectively the lines of intersection of the planes AOO' and BOO' with the plane ECG. Let the angle $BCB' = C$, $\angle POA = \alpha$, $\angle POB = \beta$, $\angle P'O'A' = \alpha'$, $\angle P'O'B' = \beta'$. We have also $\angle P'O'D' = \theta$ and $\angle POD = \rho$.

This construction represents an application of Fresnel's fundamental law that, for a biaxial mineral, the direction of extinction in any section is found by taking the intersection of the plane of the section with that which bisects the angle between the two planes respectively containing an optic axis and the line (ray of light), perpendicular to the section.

It follows that

$$\rho = \alpha - \frac{\alpha + \beta}{2} = \frac{\alpha - \beta}{2},$$

$$\text{and} \quad \theta = \frac{\alpha' - \beta'}{2}.$$

$$\tan \alpha = \frac{\tan \alpha'}{\cos C};$$

$$\tan \beta = \frac{\tan \beta'}{\cos C};$$

$$\tan 2\rho = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}; \tag{1}$$

$$\tan 2\theta = \frac{(\tan \alpha' - \tan \beta') \cos C}{1 + \tan \alpha' \tan \beta' \cos^2 C}. \tag{2}$$

Equation (2) is a special case of Michel Lévy's general formula on page 65 of the "Minéralogie Micrographique," and also of the new formula by Cesàro for expressing the extinction on any face of a zone.* The mathematical treatment of these expressions shows that, as the section moves continuously from the position (010) to the position (100), it will pass through an intermediate position of maximum extinction, if the angle $2V$ be less than 90°, or the angle $(\alpha + \beta)$ greater than 90°. Thus, θ has a maximum when $\cos c = \frac{1}{\sqrt{\tan \alpha \tan \beta}}$.† This is true of

* Mémoires de l'Académie Royale des Sciences, etc. de Belgique, 1895, Vol. LIV. p. 26. Ref. in Zeit. für Kryst., etc., 1897, p. 181.

† The position of the section where an extinction may be observed between

all amphiboles so far as known, except pargasite and a very few other varieties; with these exceptions, each has a maximum in a section far removed from the plane of symmetry. A striking example is to be found in an interesting amphibole rich in ferrous oxide from Philipstad, Sweden. It has a pronounced zonal structure; all the zones extinguish together in (010) at 15° 9′, but at different angles on (110), the latter varying from 21° to 17°, corresponding to different and unusually small optic angles in the respective zones of from 50° to 60°.

Plates I., II., and III. represent diagrammatically the variation which may be observed in the behavior of certain negative amphiboles that have such maxima of extinction, — namely, those with optical angles of 50°, 60°, 70°, and 80°, and each characterized by extinctions on (010) of 10° (Plate I.), 15° (Plate II.), and 20° (Plate III.). The abscissa represents the angle of rotation of the section out of the plane of symmetry, the ordinate indicates the corresponding angle of extinction. Arrowheads show which plane of the vertical zone possesses the maximum extinction peculiar to each curve, and also the value of that maximum. The diagrams clearly show that the maximum extinction observable in a rock-slide examined for one of these amphiboles would be far from representing the extinction-angle on (010), and data regarding extinctions and pleochroism derived from the study of thin sections would be worthless, if not controlled by this principle. For the sake of comparison, the analogous curves for amphiboles with $2V = 90°$ and ρ respectively equal to 10°, 15°, and 20°, appear in the plates. It will be seen that there is no position of maximum extinction between (010) and (100).*

Now, among the possible positions of the movable plane, there is one which surpasses all others in interest except that of the plane of sym-

(010) and (100) just equal to that on (010) may be found from the following unpublished formula by Dr. A. C. Lane of Houghton, Mich.: —

$$\sin x = \frac{1 - \cos^2 \rho - \cos^2 V}{\cos^2 V - \cos^2 \rho},$$

where x is the angle made with (100) by the required section.

* It is characteristic of all the curves, that the angle of extinction changes very slowly in passing out from (010). This is important in the study of rock-slides, since a section may be removed several degrees (even 30° when the optical angle is large) out of the plane of symmetry, and but small error would be made in using its value of extinction as equivalent to that on (010). It would, in that case, be only necessary to be sure that the section is really in the vertical zone, as ascertained by the parallelism of cleavage cracks. That it is near the position (010) can, of course, be proved by the absence of a well defined hyperbola in convergent light.

TABLE I.

Extinction-angle on (010).	$2V = 50°$.		$2V = 60°$.		$2V = 70°$.		$2V = 80°$.		$2V = 90°$.	
	$C = 62\frac{1}{4}°$.	$C = 77\frac{1}{4}°$.	$C = 62\frac{1}{4}°$.	$C = 77\frac{1}{4}°$.	$C = 62\frac{1}{4}°$.	$C = 77\frac{1}{4}°$.	$C = 62\frac{1}{4}°$.	$C = 77\frac{1}{4}°$.	$C = 62\frac{1}{4}°$.	$C = 77\frac{1}{4}°$.
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
2	2 37	2 2	2 16	1 33	1 58	1 13	1 43	1 0	1 32	51
3	3 55	3 3	3 23	2 19	2 57	1 50	2 35	1 30	2 18	1 16
4	5 14	4 6	4 32	3 7	3 56	2 28	3 27	2 1	3 4	1 42
5	6 33	5 9	5 40	3 55	4 56	3 6	4 20	2 31	3 51	2 7
6	7 52	6 15	6 49	4 44	5 56	3 44	5 12	3 3	4 37	2 33
7	9 12	7 22	7 58	5 34	6 56	4 23	6 5	3 34	5 24	3 0
8	10 32	8 32	9 8	6 26	7 57	5 3	6 58	4 6	6 11	3 26
9	11 53	9 44	10 18	7 20	8 58	5 44	7 52	4 39	6 59	3 54
10	13 14	11 0	11 29	8 15	10 0	6 26	8 46	5 13	7 47	4 21
11	14 36	12 19	12 40	9 12	11 2	7 10	9 41	5 47	8 36	4 50
12	15 58	13 42	13 52	10 12	12 5	7 55	10 37	6 23	9 24	5 18
13	17 20	15 9	15 5	11 15	13 9	8 42	11 32	6 59	10 14	5 48
14	18 44	16 42	16 19	12 21	14 14	9 31	12 29	7 37	11 4	6 18
15	20 7	18 19	17 33	13 31	15 19	10 22	13 27	8 16	11 55	6 50
16	21 31	20 2	18 49	14 45	16 26	11 16	14 25	8 57	12 47	7 22
17	22 56	21 52	20 5	16 3	17 33	12 12	15 25	9 39	13 39	7 56
18	24 21	23 48	21 22	17 25	18 41	13 11	16 25	10 24	14 32	8 30
19	25 46	25 45	22 39	18 53	19 50	14 14	17 26	11 10	15 26	9 6
20	27 12	27 58	23 58	20 27	21 1	15 21	18 28	11 59	16 21	9 44
21	28 37	30 12	25 17	22 6	22 12	16 31	19 31	12 50	17 17	10 23
22	30 3	32 30	26 36	23 52	23 24	17 47	20 36	13 45	18 14	11 4
23	31 29	34 53	27 57	25 44	24 37	19 7	21 41	14 42	19 12	11 47
24	32 54	37 18	29 18	27 43	25 51	20 32	22 47	15 43	20 11	12 32
25	indet.	indet.	30 39	29 48	27 7	22 2	23 55	16 48	21 11	13 19

metry itself. I refer to the plane of prismatic cleavage. Table I. contains the results of calculations (repeated applications of equation 2), intended to find the value of θ, (θ'), for amphiboles when C is equal to half the cleavage angle (taken at $62\frac{1}{4}°$), $2V$ at successive values of 50°, 60°, 70°, 80°, and 90°, and ρ with all values of whole degrees from 2° to 25° inclusive. Thus, if any two of the three terms $2V$, ρ, or θ' be known, the third can be determined. The values of θ' are tabulated in the col-

umn headed "62½°." This table is regarded as capable of more general application than the one published by Harker,* inasmuch as his refers only to that small class of amphiboles which have the acute optical angle situated about the axis of elasticity lying next to the vertical axis (optically positive, represented by pargasite). It will be observed that for amphiboles with small optical angle (up to 60°), any one of the three variables may be determined if the other two are known, but that, considering the instrumental errors of reading, the larger values of the optical angle cannot with much safety be determined on account of the slow variation in the corresponding angles of extinction on (010) and (110). When $2V$ is equal to 70°, ρ and θ' are for any negative species nearly equal.

The following is the similar table (Table II.) for pyroxenes, where the cleavage angle is taken at 92° 54′. It is simply a slight extension of that of Harker.†

TABLE II.

Angle of extinction on (010).	Values of θ'.			Angle of extinction on (010).	Values of θ'.		
	$2V = 50°$	$2V = 60°$	$2V = 70°$		$2V = 50°$	$2V = 60°$	$2V = 70°$
°	°	°	°	°	°	°	°
35	29¼	30½	31½	45	39	40¼	41¾
36	30¼	31¼	32¾	46	40	41¼	42¾
37	31¼	32¼	33¾	47	41	42¼	44
38	32¼	33¼	34¾	48	42	43¼	45
39	33¼	34¼	35¾	49	43¼	44¼	46
40	34¼	35¼	36¾	50	44¼	45¼	47
41	35	36¼	37¾	51	45¼	46¼	48
42	36	37¼	38¾	52	46¼	47¾	49¼
43	37	38¼	39¾	53	47¼	48¾	50¼
44	38	39¼	40¾	54	48½	49¾	51¼

We are not yet, however, in a position to make universal use of cleavage pieces for the purpose of finding the value of ρ. There are many

* Extinction-angles on cleavage-flakes. Mineralogical Magazine, 1894, Vol. X. p. 239.

† *Op. cit.*, p. 240.

rock-forming pyroxenes and amphiboles which, owing to the small size of the crystals or their very friable nature, it is extremely difficult, if not impossible, to cut in the directions necessary to obtain ρ and $2V$. On the other hand, good cleavage flakes are almost always to be had, and it is by the use of these that I propose the following method of finding the extinction on the clinopinacoid of an amphibole or pyroxene.

A perfectly flat cleavage piece, thick enough to give the greatest possible definiteness to the position of extinction and showing clearly marked cleavage cracks, is laid on an object-glass with the broadest face down. It is then carefully mounted on the stage of a two-circle Fedoroff table. With the vertical circle set at zero, the stage of the table is turned so as to bring the cleavage cracks of the specimen into a position parallel to the axis of the vertical circle.* This axis should be parallel to the principal section of either polarizer or analyzer. By taking the average of a number of good readings, the extinction angle is now obtained. Following this operation, the vertical circle is turned in such a direction that the plane of symmetry of the crystal is *more* oblique to the polarized ray and by an angle nearly approaching that at which the specimen would begin to slide on the object glass. I have found that 15° is a convenient amount of rotation, and that angle will be used in the following discussion. Extinction is again read in this new orientation with the greatest possible care.

We have in this way determined two special angles of extinction (θ' and θ''), corresponding to two planes in the vertical zone, which are at different angles (C' and $C'' = C' + 15°$) to (010). It is now possible in a simple way to eliminate a and β, and thus permit of the determination of ρ directly from θ' and θ''. To this end, we have the following series of transformations, which I owe to Mr. J. K. Whittemore of Harvard University.

Substituting in (2) the special values of θ and C, we have,

$$\tan 2\theta' = \frac{(\tan a - \tan \beta)\cos C'}{1 + \tan a \tan \beta \cos^2 C'}; \quad \text{and} \tag{3}$$

$$\tan 2\theta'' = \frac{(\tan a - \tan \beta)\cos C''}{1 + \tan a \tan \beta \cos^2 C''}. \tag{4}$$

Then

$$\frac{\tan 2\rho}{\tan 2\theta'} = \frac{1 + \tan a \tan \beta \cos^2 C'}{(1 + \tan a \tan \beta)\cos C'} = u. \tag{5}$$

* The Nachet form of the table or the simpler model by Fuess is best for the purpose.

$$(6) \qquad \frac{\tan 2\rho}{\tan 2\theta''} = \frac{1 + \tan\alpha \tan\beta \cos^2 C''}{(1 + \tan\alpha \tan\beta)\cos C'''} = v.$$

$$(7) \qquad \tan\alpha \tan\beta = \frac{1 - u\cos C'}{u\cos C' - \cos^2 C'} = \frac{1 - v\cos C''}{v\cos C'' - \cos^2 C''}.$$

$$(8) \quad \frac{\tan 2\theta' - \cos C' \tan 2\rho}{\tan 2\rho \cos C' - \cos^2 C' \tan 2\theta'} = \frac{\tan 2\theta'' - \cos C'' \tan 2\rho}{\tan 2\rho \cos C'' - \cos^2 C'' \tan 2\theta''}.$$

Let $\tan 2\rho = x$, $\tan 2\theta' = x'$, $\tan 2\theta'' = x''$. We have

$$(9) \qquad \frac{x' - x\cos C'}{x\cos C' - x'\cos^2 C'} = \frac{x'' - x\cos C''}{x\cos C'' - x''\cos^2 C''}.$$

Hence

$$(10) \quad x(x'\cos C''\sin^2 C' - x''\cos C'\sin^2 C'') = x'x''(\cos^2 C'' - \cos^2 C').$$

Substituting the values of x, x', and x'',

$$(11) \qquad \tan 2\rho = \frac{\tan 2\theta' \tan 2\theta'' (\cos^2 C'' - \cos^2 C')}{\tan 2\theta' \cos C'' \sin^2 C' - \tan 2\theta'' \cos C' \sin^2 C''}.$$

Reducing to a form more convenient for the use of logarithms, we have

$$(12) \qquad \tan 2\rho = \frac{\tan 2\theta' \tan 2\theta'' \sin(C' + C'')\sin(C' - C'')}{\tan 2\theta' \sin^2 C' \cos C'' - \tan 2\theta'' \sin^2 C'' \cos C'}.$$

We now have an expression by means of which the extinction can be calculated even when the optical angle is not known. The theory of the same problem was elaborated by Liebisch in his discussion* of the general determination of the optical angle and of the direction of the optic axes in biaxial minerals, using sections whose crystallographic orientation is known, and on which the planes of vibration of the two refracted rays are known. Of immediate interest to us is the case in the monoclinic system where the optical plane is also the plane of symmetry, but the orientation of the two axes of elasticity is not determined. He finds the problem soluble when two sections can be employed, and this is what we practically have for optical purposes in the cleavage piece placed at two positions with respect to the incident ray from the polarizer.

But it is evident that there must be some degree of error in the instrumental readings of θ' and θ''; we have next to inquire if equation (12) is so sensitive to changes in θ' or θ'' or to simultaneous changes in both as to make any determinations of ρ by means of it valueless. By ac-

* Über die Bestimmung der optischen Axen durch Beobachtung der Schwingsrichtung ebener Wellen. Neues Jahrbuch für Min., etc., 1886, Bd. I. p. 155.

tually trying certain cases, I have found that ρ changes for small errors in θ' and θ'', but that the rate of change is not sufficiently rapid to make the method of no practical use. Indeed, the errors in ρ may in certain cases be little greater than the original errors of reading θ' and θ''. A Zillerthal actinolite may be taken as an example.

Let $C' = 62° 15'$, and $2V = 80°$ (an actinolite). Then $\theta' = 13° 27'$; and, if $C'' = 77° 15'$ (15° of rotation of the vertical circle), then $\theta'' = 8° 16'$. In the attempt to find ρ from θ' and θ'', errors of various magnitudes in θ' and θ'' were introduced as follows.

TABLE III.

	$\theta' = 13° 27'$ + Error of	$\theta'' = 8° 16'$ + Error of	ρ determined as	Error in ρ.
1	0°	+10′	14° 31′	−29′
2	0°	+20′	14° 3′	−57′
3	0°	+30′	13° 39′	− 1° 21′
4	+10′	+10′	15° ½′	+ ½′
5	+20′	+20′	15° 2′	+ 2′
6	+30′	+30′	15° 5′	+ 5′
7	+ 1°	+ 1°	15° 11′	+11′
8	+10′	−10′	16° 8′	+ 1° 8′
9	+20′	−20′	17° 28′	+ 1° 28′
10	+30	−30′	19° 2′	+ 2° 2′

The limit of error in reading extinction-angles for colorless cleavage flakes, or for those that do not display very deep absorption, may be placed at 20′. At this limit, in our example, there is seen to be an error of rather less than 1° for an error in θ'', with θ' exactly determined. When θ' and θ'' show errors of reading in the *same* direction (in the example, both plus), they may be large without making an equally large error in ρ. This is important, since any fault in the construction of the microscope will thus equally affect both readings and need not greatly influence the value of ρ, even though found by means of so sensitive an

equation. On the other hand, if θ' and θ'' vary from the truth in opposite directions, ρ quickly changes, a fact which is evident from an inspection of the expression for tan 2ρ. The table shows an error of 1° 28′ in ρ when θ' is 20′ too large and θ'' 20′ too small; and again an error of +2° 2′ in ρ for corresponding errors of +30′ and −30′ in θ' and θ''.

If the rotation of the vertical circle had been in the opposite direction through the same angle, so as to make $C' = 47° 15'$, the errors for cases 1, 2, 3, 8, 9, and 10 of the foregoing table would have been considerably less; those for cases 4, 5, 6, and 7, on the contrary, somewhat greater. The curves of Plates I., II., and III. show, however, that the extinction angles for each of the different amphiboles in sections cut at respective angles of 47° 15′ and 62° 15′ to the plane of symmetry would be nearly the same, and that the variation in the extinction-angle at 47° 15′ in passing from one amphibole to another would be much slower than that peculiar to the 77° 15′ position.* Hence I have chosen the latter as the more useful; in Table I., in the columns headed "$C = 77° 15'$," will be found the values of extinction angles characteristic of the same amphiboles whose extinctions on cleavage pieces have already been calculated. By the use of the whole table, a first approximation to the value of the extinction-angle on (010) can be rapidly made without the necessity of going through the rather tedious application of equation (12).

Analogous results characterize the introduction of errors into the θ' and θ'' of pyroxenes. I have chosen an Ala diopside with $C' = 43° 33'$, $2V = 59°$, and $\rho = 36°$. Then, revolving the cleavage-face (110) 15° away from (010), we have $C'' = 58° 33'$. θ' was calculated to be 31° 16′, and $\theta'' = 26° 8'$. Introducing arbitrary errors in θ' and θ'', we obtain the results of the accompanying Table IV.

Generalizing from the two error tables, supplemented by the inspection of equation (12), we can reach certain conclusions regarding the influence of instrumental errors. Equation (12) is least sensitive for errors in θ' and θ'' when these are either both plus or both minus and equal or nearly equal. When equal, ρ can be more accurately determined than by direct measurement on a section in the plane of symmetry (given the use of the same microscope in both cases, as well as equal thicknesses, absorption, etc. for the cleavage piece and cut section).

* It is, of course, evident that *both* readings (at 47½° and 77½°) can be taken on the same cleavage piece; and also that amounts of rotation other than 15° may be advantageously employed. Experiment shows that the cleavage piece will not slide on the object-glass even at an angle of 20°, and thus θ may be determined for the 42½° and 82½° positions.

TABLE IV.

	$\theta' = 31° 16'$ + Error of	$\theta'' = 26° 8'$ + Error of	ρ determined as	Error in ρ.
1	0°	10′	35° 46′	−14′
2	0°	20′	35° 32′	−28′
3	0°	30′	35° 18′	−42′
4	10′	10′	36° 8′	+ 8′
5	20′	20′	36° 18′	+18′
6	30′	30′	36° 27′	+27′
7	1°	1°	36° 57′	+57′
8	10′	−10′	36° 36′	+36′
9	20′	−20′	37° 15′	+1° 15′
10	30′	−30′	37° 52′	+1° 52′

When the errors of θ and θ'' are unequal, the resulting error in ρ is more significant and is most unfavorable when the errors of reading the extinction in the two positions are of opposite sign. When equal numerically, the error in ρ for actinolite and diopside is about four times that made by directly reading the extinction on a plate cut parallel to (010). The necessity for accurate measurement of θ' and θ'' is evident. In actual practice, moreover, several independent determinations of ρ should be carried out and an average taken to secure the safest results.

Two examples may suffice to show the kind of results obtained by the author in the application of the method here outlined. The Philipstad hornblende already referred to was studied with reference to the extinction on (010), first by the use of two carefully oriented sections made by M. Werlein of Paris, then by the use of cleavage pieces. The former gave an extinction on (010) of 15° 5′ (white light). Extinction was then read on a cleavage flake and found to be 20° 53′, as the result of averaging many readings. This flake was then turned into the position where the plane perpendicular to the ray of light from the polarizer made an angle of $77\frac{1}{2}°$ with the plane of symmetry of the crystal. Ex-

tinction was read in this position against the cleavage trace, at an angle with the latter of just 20° (average of ten readings). Substituting this value and the value of extinction on (110) in equation (12), I obtained 15° 3′ as the corresponding value of ρ. This very close correspondence with the determination of ρ by the use of the oriented sections is not less than accidental, but the example clearly shows that cleavage pieces may be made to yield information concerning the value of extinction on the plane of symmetry as useful for practical purposes as that derived from a section in that plane.

A second example was found in a hornblende, which was given me for examination by Professor J. E. Wolff of Harvard University. It occurs associated with much pyroxene and biotite in the classic coarse theralite of Theralite Peak in the Crazy Mountains. Optical study of the hornblende in the numerous slides which have been made of the rock is difficult, on account of the scarcity of the mineral in large individuals. It would be quite impossible to make oriented sections even if the grains could be freed from the much more abundant pyroxene, with which they are commonly intergrown. Apart from the absolute small amount of the hornblende present in the rock and from the fact of intergrowth with another silicate, this seemed to be a particularly unfavorable case for the application of our method, in that the absorption is strong, and errors in θ' and θ'' should appear rather larger than characterize readings on such an amphibole as Zillerthal actinolite, for example. The rock was pulverized and, after some search, suitable cleavage pieces were discovered in the powder, and manipulated as in the last example. This time, the curve of extinctions was examined on four points besides that at the position (010); namely, at the positions, 42¼°, 47¼°, 77¼°, and 82¼°. Extinctions were read for these at respective values of 29°, 29° 30′, 31°, and 30°. The cleavage position gave 34°. The average value of ρ now determined by substituting the readings in the general equation is 28° 5′. Now, I was fortunate enough to find in one of the thin sections a longitudinal section of the hornblende, evidently in the vertical zone since the cleavage cracks were rigorously parallel to one another, and very near the plane (010), inasmuch as there was practically no trace of a hyperbola in convergent light. Careful reading of the extinction afforded an astonishingly close approximation to the value of ρ just determined, viz. 28°. It is possible that the true angle is half a degree or more greater or less than that, but the calculated value is in any case near that observed in the rock-slide. It may be noted in passing that this is a rather remarkable hornblende, from the fact that its extinction angle is

abnormally high; the optical angle is also unusual, for calculation shows that it must be about 55°. Unfortunately, for reasons specified above, the mineral cannot be separated and chemically analyzed.

In the process of working out these illustrations, the author has come to the conclusion that, with the proper safeguards, the method can be safely applied to the amphiboles in general; and that the pyroxenes can be similarly treated. It may thus prove to have more than mere theoretical interest.

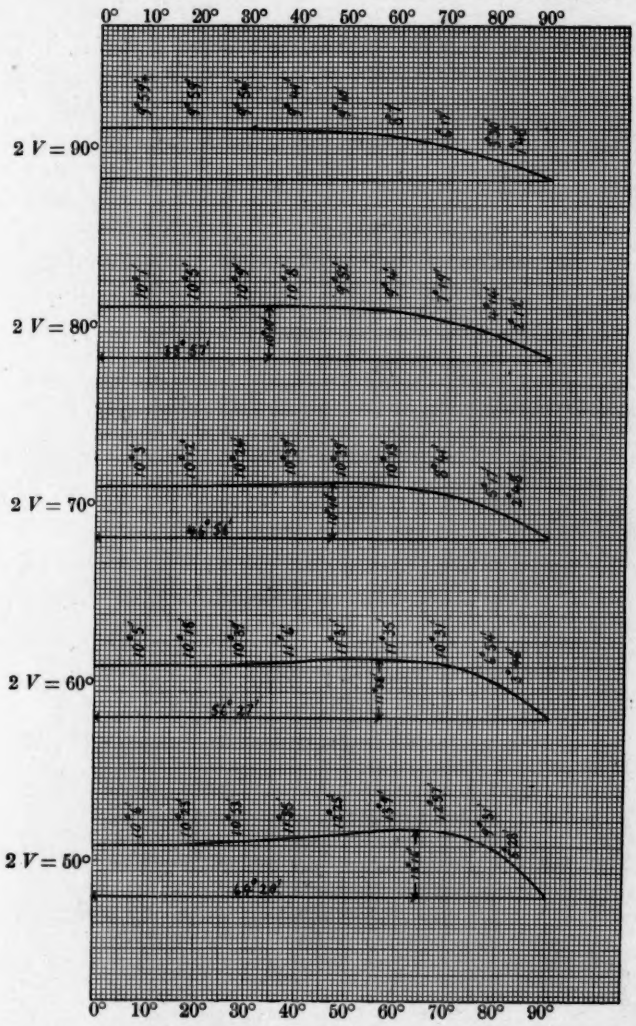

PLATE I. — Showing the curves of extinction in the vertical zone of Amphiboles for which the extinction on (010) is 10° and the optical angle is 50°, 60°, 70°, 80°, or 90°. The abscissa represents the amount of rotation of a plane in the zone away from the plane of symmetry. The ordinate is the corresponding angle of extinction for that position of the movable plane. The position and amount of maximum extinction are represented by arrow-heads. The values of extinction for 10° intervals in the zone are entered in the diagram.

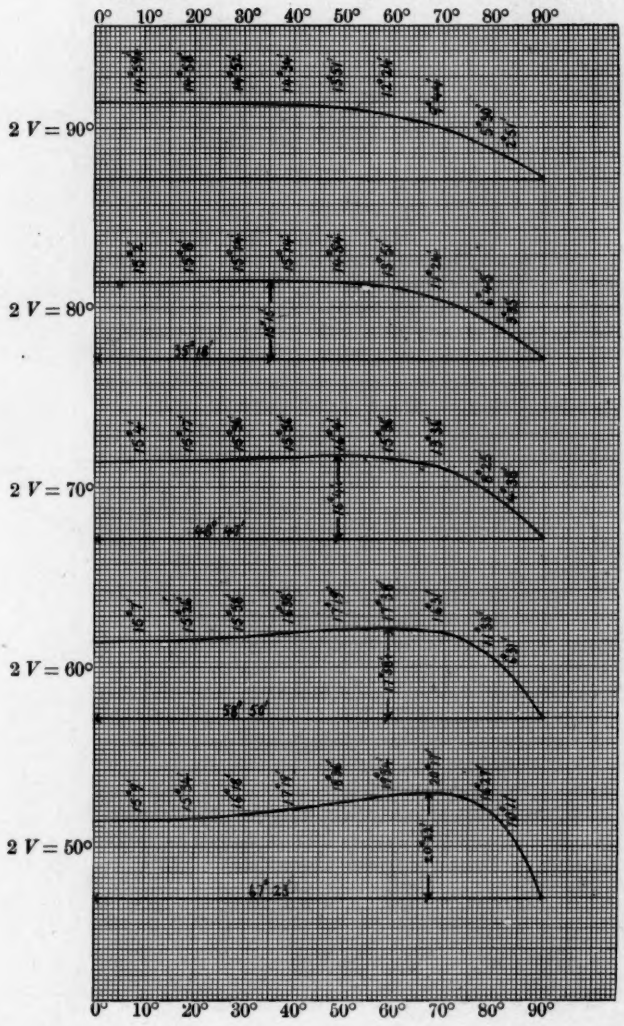

PLATE II. — Showing curves formed on the same plan as those of Plate I. The extinction on (010) is here 15°.

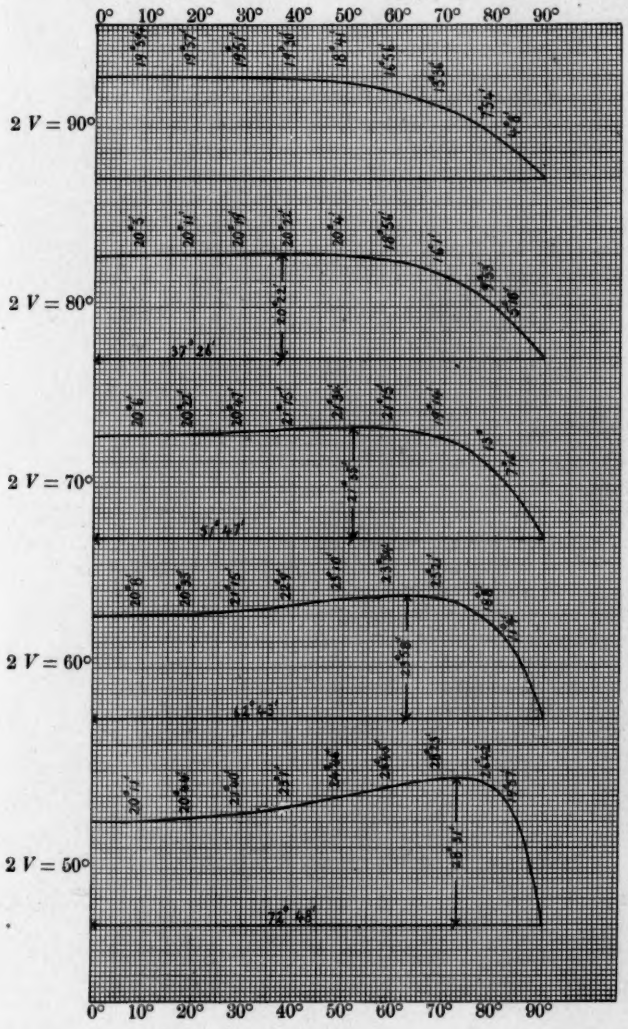

PLATE III. — Showing curves formed on the same plan as those of Plate I. The extinction on (010) is here 20°.